New parity-violating photon-axion interaction

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A variant of an earlier proposal by the author and SenGupta, to describe four dimensional Maxwell electrodynamics in Einstein-Cartan spacetimes through a Kalb-Ramond field as an intermediary, is shown to lead to a new Maxwell-Kalb-Ramond coupling that violates spatial parity, even when the KR gauge field has its standard parity assignment. One consequence of this coupling seems to be a modulation, independent of wavelength but dependent on the KR field strength, of the intensity of synchrotron radiation observed from distant galactic sources.

The problem of describing the four dimensional Maxwell field in an Einstein-Cartan (EC) spacetime, in a manner that explicitly preserves electromagnetic $U(1)$ gauge invariance, has been addressed, for the special case of a completely antisymmetric torsion [1], through the introduction of an antisymmetric second rank tensor gauge potential $B_{\mu\nu}$ also known as the Kalb Ramond (KR) field. In the approach of ref. [1], the KR field strength tensor of this gauge potential $H_{\mu\nu\lambda} \equiv \partial_{[\mu} B_{\nu\lambda]}$ is augmented by a $U(1)$ Chern-Simons form, in accord with certain consistency requirements of perturbative heterotic string theory [2], as

$$H_{\mu\nu\lambda} \to \tilde{H}_{\mu\nu\lambda} \ \equiv \ H_{\mu\nu\lambda} \ + \ \frac{1}{3}\sqrt{G} A_{[\mu} F_{\nu\lambda]} \ , \tag{1}$$

where, G is Newton's constant. This augmented KR field strength is then made to couple to torsion by means of a simple contact interaction, so that the ECKR-Maxwell action is

$$\begin{aligned} S \ = \ & \int d^4x \ \sqrt{-g} \ [\ \frac{1}{16\pi G} \ R \ (g \ , \ T) \ - \ \frac{1}{4} F_{\mu\nu} \ F^{\mu\nu} \ - \frac{1}{2} \ \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} \\ & + \ \frac{1}{\sqrt{G}} \ T^{\mu\nu\lambda} \ \tilde{H}_{\mu\nu\lambda} \ ] \end{aligned} \tag{2}$$

where R is the scalar curvature. The torsion tensor $T_{\mu\nu\lambda}$ now plays the role of an auxiliary field in eq. (2), obeying the constraint equation

$$T_{\mu\nu\lambda} \ = \ \sqrt{G} \ \tilde{H}_{\mu\nu\lambda} \ . \tag{3}$$

Thus, the augmented KR field strength three tensor plays the role of the spin angular momentum density which is the source of torsion [3]. In fact, (3) can be used to eliminate the torsion from the theory and yield an 'on-shell' action in which the Maxwell-KR interaction, to leading order in the Planck length $\sqrt{G}$ is given by

$$S_{int} \ = \ \sqrt{G} \ \int \ d^4x \ \sqrt{-g} \ H_{\mu\nu\rho} A^{[\mu} F^{\nu\rho]} \ . \tag{4}$$

Now, the KR three tensor is Hodge-dual to the derivative of a spinless field H, so that, after a partial integration, one obtains,

$$S_{int} \ = \ \frac{1}{2} \ \sqrt{G} \ \int d^4x \ H \ F_{\mu\nu} \ {}^*F^{\mu\nu} \ , \tag{5}$$

where, ${}^*F^{\mu\nu} \ \equiv \ \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$. Here, we have noted the fact that

$$\frac{1}{\sqrt{-g}} \partial_\nu \ (\sqrt{-g} \ {}^*F^{\mu\nu}) \ = \ D^S_\mu \ {}^*F^{\mu\nu} \ = \ 0 \tag{6}$$

by the Maxwell Bianchi identity, where, D^S is the covariant derivative using the Christoffel connection.

The consequences of this interaction between the Maxwell and KR fields have been explored [4] within the context of synchrotron radiation from cosmologically distant radio galaxies and quasars. The most important effect appears to be an optical activity in the form of a rotation of the plane of polarization of the radiation through an angle that is proportional to the time rate of change of the axion field that is Hodge-dual to the KR field strength [4]. This rotation is independent of wavelength for small wavelengths, and in that is to be contrasted with Faraday rotation. Another feature of this optical activity is its 'universality' in that it is quite independent of source properties (unlike, for instance, the Faraday rotation measure), although it does depend on the redshift of the source. In the string-inspired situation, the KR field $B_{\mu\nu}$ is an even spatial parity tensor so that the spinless H field is a *pseudo*scalar - the axion. It is obvious in this case that the interaction in (5) preserves spatial parity. However, one can consider the case where $B_{\mu\nu}$ has odd parity so that H is an even parity scalar; the interaction (5) in that case violates spatial parity. This latter situation is interesting from the point of view of polarization anisotropies of the Cosmic Microwave Background Radiation (CMBR): certain odd-parity multipole moments of the polarization tensor turn out to be non-vanishing if such an interaction is present [5]. It is also interesting for its implications in high energy physics [6] involving, e.g., helicity-flip scattering of Dirac fermions [7].

In this paper, we study a variant of the augmentation in (1), motivated primarily by an attempt to describe tensor gauge fields (of rank two and three) in EC spacetimes by means of a framework similar to the one above.

It is not difficult to see that the minimal coupling procedure (to an EC spacetime) has the same pathology with gauge invariance for second rank ($A_{\mu\nu}$) and third rank (i.e., $A_{\mu\nu\rho}$) Abelian antisymmetric tensor gauge fields as for the Maxwell field A_μ.[1] E.g., let us take the second rank antisymmetric gauge field (a different KR field, considered as a gauge field rather than a source of torsion). If one defines the spacetime covariant field strength as

$$F_{\mu\nu\rho} \ \equiv \ D_{[\mu} \, A_{\nu\rho]} \tag{7}$$

one observes that under the tensor gauge transformation

$$\delta_\Lambda \, A_{\mu\nu} \ = \ D_{[\mu}\Lambda_{\nu]} \ , \tag{8}$$

the field strength transforms as

$$\delta_\Lambda \, F_{\mu\nu\rho} \ = \ T_{[\mu\nu}^{\ \ \eta} \, [\partial_\eta \Lambda_{|\rho]} - \partial_{\rho]}\Lambda_\eta] \ + \ `TT' \ , \tag{9}$$

where, $`TT'$ signifies terms that are quadratic in the torsion. The rhs of (9) is certainly non-vanishing and signifies a violation of tensor gauge invariance.

One may try to adapt the procedure outlined earlier to deal with this problem for rank 2 and rank 3 fields, by using an augmented KR field á la [1]. The problem is with the augmentation. Using differential forms, it is immediately obvious that the Chern-Simons augmentation $\sqrt{G} A \wedge F$ vanishes in four dimensions for gauge connection p-forms A for $p > 1$, where F is the curvature $p+1$-form corresponding to A. To construct a 3-form which one can append on to the KR 3-form, one can try instead

$$\tilde{\mathbf{H}} \ = \ \mathbf{H} \ + \ \frac{1}{3} \, \sqrt{G} A \wedge \ {}^*F \tag{10}$$

where ${}^*F$ is the Hodge-dual of the curvature $p+1$ form, and $\mathbf{H}$ is the KR 3-form (with coordinate-frame components $H_{\mu\nu\rho}$).

With the KR 3-form thus augmented, the procedure of ref. [1] outlined above can be followed to determine the coupling of the tensor gauge fields to torsion and eventually to the KR field itself. One obtains generic KR-tensor gauge field interactions, given schematically to $O(\sqrt{G})$, by

$$S_{int} \sim \sqrt{G} \ \int d^4x \ \sqrt{-g} \ H \ F \cdot F \ , \tag{11}$$

i.e., a coupling of the axion H to the kinetic energy part of the Lagrangian of the tensor gauge field. It is obvious that with the axion being a pseudoscalar, interaction terms like (11) *violate spatial parity.* While it is not clear if augmentations as in (10) arise naturally within string theories, interactions as above may have interesting observable consequences.

A particularly engaging case in point is that of the Maxwell field. Combining (1) and (10), one can define an 'improved' augmented KR field strength 3-form as

$$\tilde{\mathbf{H}} \ = \ \mathbf{H} \ + \ \frac{1}{3} \ \sqrt{G} \ [\alpha_- \ A \wedge \ {}^*F \ + \ \alpha_+ \ A \wedge \ F] \ , \tag{12}$$

where, $\alpha_\pm$ are dimensionless real numbers. Thus, one is lead to the $O(\sqrt{G})$ interaction

[1] Such gauge fields occur, albeit in higher dimensions, in the non-perturbative sector of string theories involving D-branes [8].

$$S_{int} \sim \sqrt{G} \int H\ [\alpha_-\ F \cdot F\ +\ \alpha_+\ F \cdot\ ^*F]. \tag{13}$$

With H the axion, as is the situation in heterotic string theory, for example, the first term in (13) is parity-violating and the second, parity-preserving. To emphasize, unlike in earlier work [6], [7], where one needed the KR gauge potential to have the 'wrong' (i.e., odd) parity in order to produce parity-violating effects, here it is instead an augmentation of the KR field strength suggested by an attempt to generalize the procedure to the case of higher rank tensor gauge fields, that is responsible for parity-violation.

Because of the difference in the parity behaviour between the two terms in (13), the physical consequences stemming from them are quite distinct, at least to leading order in the Planck scale. In other words, the consequences of the parity-preserving term in terms of a cosmological optical activity [4] are by no means affected by the presence of the parity-violating interaction.

To examine the consequences of the parity-violating interaction, we once again consider synchrotron radiation from distant galactic sources. Here we confine ourselves to probing the effect of this term on a plane electromagnetic wave in a Minkowskian background spacetime. The more realistic case of radiation from large redshift sources in an expanding universe will be taken up elsewhere. Thus, in this paper, the idea is to anticipate the kind of phenomenon that may occur in realistic situations due to the new interaction.

To this end, we consider the Maxwell equations to linear order in $\sqrt{G}$ as was done in ref. [4], and treat the KR field as a background whose dynamics is basically that of the free KR theory. This dynamics follows easily from the Bianchi identity that the KR field strength must be subject to due to its own gauge invariance. It implies that the axion must satisfy the massless Klein-Gordon equation $\Box H = 0$. The Maxwell equations now assume the form

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \sqrt{G}\ \nabla H \cdot [\alpha_+\ \mathbf{B}\ +\ \alpha_-\ \mathbf{E}] \\
\dot{\mathbf{E}}\ -\ \nabla \times \mathbf{B} &= -\sqrt{G}\{\alpha_+\ [\dot{H}\mathbf{B}\ -\ \nabla H \times \mathbf{E}] \\
&\quad +\ \alpha_-\ [\dot{H}\mathbf{E}\ -\ \nabla H \times \mathbf{B}]\} \\
\dot{\mathbf{B}}\ +\ \nabla \times \mathbf{E} &= 0\ =\ \nabla \cdot \mathbf{B}\ .
\end{aligned} \tag{14}$$

With potential cosmological applications in mind, we consider the simplest situation in which the axion is spatially independent, as was assumed in [4]. This implies that the axion field has a linear time dependence $H(t) = h_0 t + h_1$, with the constants h_0, h_1 being odd under parity, in accord with the intrinsic parity of the axion. The ∇H terms in (14) can now be dropped, and the resulting equations manipulated to eliminate the $\mathbf{E}$ field for example. The resulting 'wave equation' for the $\mathbf{B}$ field is

$$\ddot{\mathbf{B}}\ -\ \nabla^2 \mathbf{B}\ +\ \sqrt{G}\ h_0\ [\alpha_+\ \nabla \times \mathbf{B}\ +\ \alpha_-\ \dot{\mathbf{B}}]\ =\ 0\ . \tag{15}$$

To further simplify the notation, we rescale $h_0 \to h_0/\sqrt{G}$ and choose the ansatz

$$\mathbf{B}\ =\ \mathrm{Re}\ [\mathbf{b}(t)\ e^{i\mathbf{k}\cdot\mathbf{x}}]\ . \tag{16}$$

Defining components of right and left circular polarization $b_\pm \equiv b_x \pm i b_y$, with z as the direction of propagation, eq. (15) reduces to,

$$\ddot{b}_\pm\ +\ h_-\ \dot{b}_\pm\ +\ \left(k^2 \mp h_+\ k\right) b_\pm\ =\ 0\ , \tag{17}$$

where, $h_\pm \equiv h_0\ \alpha_\pm$. Observe that (17) reduces, for $h_- = \alpha_- = 0$, to eq. (14) of ref. [4] which leads to the optical activity already discussed. Here, the angle of rotation of the plane of polarization of the electromagnetic wave is, for large values of k, once again given by $|h_+|\ t$. The effect of parity violation is confined to the second term, which signifies a *modulation*, i.e., either an enhancement or an attenuation, of the intensity of the observed radiation, depending on the sign of h_-.

There are a few features of the new modulation effect due to the parity-violating interaction that should be commented upon.

First of all, the modulation is proportional to the time rate of change of the axion field and not just to its absolute value, unlike the rotation angle of the polarization plane discerned in [4]. From a cosmological standpoint, one expects it to be negative, because of expansion effects of the universe. There is dependence also on the undetermined coefficient α_- which occurs in the proposed augmentation (12) of the KR field strength. If $\alpha_- > 0$, then with $h_- < 0$, eq. (17) would appear to predict an *enhancement* in the intensity of radiation - a rather startling result. For $\alpha_- < 0$, in contrast, one would observe an attenuation of that intensity. This uncertainty in the physical outcome appears to be unavoidable within the somewhat phenomenological approach of this work. Recall that in the augmentation of the KR field proposed in [1], such an uncertainty in the coupling does not exist, as the coefficient is fixed essentially by consistency requirements in string theory. Here, as already remarked, one is not certain that there is such a microscopic underpinning of the proposed augmentation.

Secondly, the first derivative term in (17) and hence the modulation is *independent* of the wavelength of the radiation. This indicates a modulation of the intensity which is different, for example, from the one that seems to emerge [9] when one considers an axion field that is harmonic in its spatial dependence (rather than being spatially-independent as considered here); the damping term in that case is proportional to the ratio of the wavelength of the axion to that of the radiation field.

Thirdly, it is independent of the intrinsic properties of the galactic source emitting the radiation or indeed the intergalactic medium it traverses and in that sense 'cosmological' in character. Thus, it is unlike viscous damping caused by scattering of the wave during its propagation through galactic and intergalactic medium. In a true cosmological scenario, we anticipate the modulation to depend on the redshift of the source.

If our considerations are correct and there indeed exists a KR field producing torsion as envisaged here, and this field affects electromagnetic radiation from distant sources in the manner outlined above, then one should observe these effects in planned experiments on the CMBR. We postpone a detailed discussion of the precise nature of such an effect to a future publication. What is however interesting at this stage of our investigation is the existence of two seemingly independent means of observing a 'primordial' KR field (and hence of torsion), one of which is also a signal of primordial parity violation. Indeed, there possibly are other, perhaps more significant, phenomenological implications of this parity violation that we have not been able to identify.

Finally, just as the Chern Simons augmentation has a gravitational analogue, given by the 3-form $\omega \wedge R$ where ω is the Lorentz spin connection one-form and R is the Riemann curvature two-form, there ought to be 'parity-violating' gravitational augmentations characterised by the 3-form $\omega \wedge {}^*R$ where ${}^*R$ is the dual curvature. Thus, one anticipates new parity-violating gravitational interactions of the type

$$S_{int} \sim \int H \; R \cdot R \; . \tag{18}$$

One effect that such an interaction might produce could be a modulation of gravitational waves. If it turns out that the analogue of α_- in that case is positive, while the time rate of change of the axion is negative, one will be led to the truly remarkable prediction of enhancement of the intensity of gravitational waves from distant sources. This does seem to warrant a more thorough investigation.

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